At least vs. _more than_: Explaining ignorance in terms of relevance of speaker belief

Overview. The standard grammatical theory of scalar implicatures (SIs) (Chierchia 20004; Chierchia et al. 2012; Fox 2007) posits that SIs are derived in grammar, whereas ignorance inferences (IIs) are derived pragmatically, as quantity implicatures. Specifically, the standard theory predicts that for any utterance S and any relevant proposition ϕ which isn't entailed, and whose negation isn't entailed, by S, the maxim of quantity (MQ) licenses an inference of speaker ignorance about ϕ. Buccola & Haida (forthcoming) (henceforth *B&H*) explore a variant of the grammatical theory in which relevance is closed under belief (if ϕ is relevant, then it's also relevant whether the speaker believes ϕ). This move has the effect that IIs, like SIs, can only be derived in grammar, via a covert belief operator (Meyer 2013; Fox 2016). MQ, they show, then no longer enriches the meaning of an utterance, per se, but rather acts as a filter on what can be relevant in any context where MQ is active – a property they dub *obligatory irrelevance* (OI). In our talk, we argue that OI explains the contrast in IIs exhibited by *at least* vs. *more than*, while the standard theory requires arbitrary stipulations to prevent overgeneration of IIs.

Empirical background. Sentence (1) obligatorily implies speaker ignorance about the number of dogs Ann owns, while (2) doesn't (though it's compatible with speaker ignorance) (Geurts & Nouwen 2007; Nouwen 2010, 2015). Specifically, (1) implies that the speaker is ignorant about whether Ann owns exactly two dogs or more than two dogs (see also Büring 2008; Schwarz 2016), while (2) doesn't.

(1) Ann owns at least two dogs.

(2) Ann owns more than two dogs.

To be more precise, we characterize (speaker) ignorance about a proposition ϕ as not knowing ϕ ($\neg K\phi$) and not knowing the negation of ϕ ($\neg K\neg\phi$), the conjunction of which we abbreviate as $I\phi$. Thus, (1), whose denotation we write as $[\geq 2]$, implies $I[=2]$ (ignorance about exactly two) and $I[>2]$ (ignorance about more than two), while (2), $[>2]$, implies neither $I[=3]$ nor $I[>3]$.

Standard grammatical theory. A sentence S may be parsed as *exh* S, with an exhaustivity operator, *exh*. What *exh* S means is that S is true and that every innocently excludable (IE) alternative of S is false (Fox 2007), where an *alternative* of S is taken to be any formal alternative (e.g. in the sense of Katzir 2007) that is relevant. In addition, the standard theory assumes the basic maxims of quality, (The speaker should only utter sentences that the speaker believes to be true), and quantity (**MQ**), (The speaker should utter a sentence S such that, for every relevant proposition ϕ that the speaker believes to be true, S entails ϕ), and that relevance is closed under conjunction and negation (if ϕ and ψ are both relevant, then so are $\phi \wedge \psi$ and $\neg\phi$) (Fox 2007).

Too many ignorance inferences. We now show that the standard theory fails to capture the contrast between (1) vs. (2). We start by establishing the generalization in (3), where S *settles* ϕ iff $[\![S]\!]$ entails ϕ or $[\![S]\!]$ entails $\neg\phi$ (Fox 2016).

(3) **Generalization 1.** If MQ is active in a context c, and if ϕ is relevant in c and S doesn't settle ϕ, then S gives rise to the inference $I\phi$. *Proof.* Suppose that MQ is active, that ϕ is relevant, and that S entails neither ϕ nor $\neg\phi$. Then MQ licenses the inferences $\neg K\phi$ and $\neg K\neg\phi$, i.e. $I\phi$.

Next, we assume that (1) and (2) have all *at least n*, *more than n*, and *exactly n* sentences as formal alternatives, for $n \in \mathbb{Q}$. That is, for our demonstration we adopt the universal density of measurement (**UDM**) hypothesis, viz. that measurement scales needed for natural language semantics are always dense (Fox & Hackl 2006), since otherwise the standard theory derives a SI for (2). If (1) is parsed with matrix *exh*, then no alternatives for $n > 2$ are IE, while those that are IE are already excluded by the meaning of the prejacent; thus, *exh* would apply vacuously, and so (1) simply denotes $[\geq 2]$ (cf. Schwarz 2016). If (2) is parsed with matrix *exh*, then, because of the UDM, *all* alternatives are IE (since there simply is no maximal set of excludable alternatives), hence are all excluded, yielding a contradiction; thus, (2) can't be parsed with *exh* (cf. Fox & Hackl 2006), and so (2) just denotes $[>2]$.

With this established, we now see that (1), $[\geq 2]$, fails to settle many presumably relevant alternatives, including not just $[=2]$ and $[>2]$, but in fact all propositions $[\geq n]$, $[>n]$, and $[=n]$ for $n > 2$. Given (3), (1) should thus license the inferences $I[\geq n]$, $I[>n]$, and $I[=n]$ for $n > 2$, contra fact. Similarly, (2), $[>2]$, fails to settle many presumably relevant alternatives, hence should license many IIs, contra fact. In short, the theory predicts far too many IIs across the board, and at first glance, provides little hope of explaining the contrast in IIs between (1) and (2), short of stipulating that all $[\geq n]$, $[>n]$, and $[=n]$ for $n > 2$ are irrelevant to (1) and to (2).

Relevance of belief. B&H add to to the closure conditions on relevance the condition that if ϕ is relevant, then it's also relevant whether the speaker believes ϕ.

(4) **Closure of relevance.** If ϕ and ψ are both relevant, then so is $\phi \wedge \psi$, $\neg\phi$, and $K\phi$.

A consequence of this move, together with MQ, is that ignorance can only be derived in grammar (Fox 2016) (proof omitted here), which means that grammar must make available a covert belief operator, K (Meyer 2013). More generally, closing relevance under belief leads to the following new generalization.

(5) **Generalization 2.** If MQ is active in c, and if ϕ is relevant in c and S doesn't settle ϕ, then $[\![S]\!] \vDash I\phi$.
Proof. Omitted here.

The difference between (3) and (5) – the effect of closing relevance under belief – has to do with how S "gives rise to ignorance". Previously, IIs were purely pragmatic, licensed directly by MQ. Now, IIs are entailments of the semantic meanings of uttered sentences. As a consequence of (5), B&H can state a precise condition on relevance in contexts where MQ is active.

(6) **Condition on relevance (consequence of closure under belief).** If MQ is active in c, and if S doesn't settle or entail ignorance about ϕ, then ϕ isn't relevant in c.

This condition restricts what propositions can be relevant in a specific context in which MQ is active. If a proposition can't be relevant (to a certain uttered sentence) in *any* context in which MQ is active, B&H call that proposition *obligatorily irrelevant*.

(7) **Obligatory irrelevance (definition).** ϕ is obligatorily irrelevant (OI) to S iff for every context c, if MQ is active in c, then S doesn't settle or entail ignorance about ϕ.

(1) and (2) revisited. We now show that closing relevance under belief captures the contrast between (1) vs. (2). Our main point is that the alternatives that previously led to unattested IIs are now OI, hence don't yield those unattested IIs.

(8) LF of (1): exh K (exh) [. . . at least two . . .] (9) LF of (2): exh K (exh) [. . . more than two . . .]

We start with (1), whose LF is given in (8). The prejacent of (the matrix) *exh* denotes simply $K[\geq 2]$. We assume as before that the formal alternatives of $[\geq 2]$ are all $[\geq n]$, $[= n]$, and $[> n]$, for $n \in \mathbb{Q}$. Crucially, all such alternatives for $n > 2$ are OI, since if we assume they're relevant in a context c, MQ can't be active in c because (8) doesn't settle or entail ignorance about those alternatives. For instance, consider $[= 7]$: even if we take as many formal alternatives to be relevant as possible, (8) won't settle $[= 7]$, and it will only entail $\neg K[= 7]$, not $\neg K\neg[= 7]$, hence won't entail $I[= 7]$; so $[= 7]$ is OI. However, $[= 2]$ and $[> 2]$ can both be relevant, and when they are, matrix *exh* delivers $\neg K[= 2]$ and $\neg K[> 2]$, which together with the prejacent, $K[\geq 2]$, entails $I[= 2]$ and $I[> 2]$ – precisely the attested IIs. (As before, an inner *exh* would be vacuous.) We move next to (2), whose LF is given in (9). We assume as before that the formal alternatives of $[> 2]$ are all $[> n]$, $[\geq n]$, and $[= n]$ for $n \in \mathbb{Q}$. Crucially, all such alternatives for $n > 2$ are again OI. For instance, $[= 7]$ is OI because (9) can't possibly settle or entail ignorance about $[= 7]$. As such, (9) just denotes $K[> 2]$, with no IIs.

Discussion. Closing relevance under belief has been argued for elsewhere on independent grounds (Fox 2016). We've shown that doing so leads to a principled restriction on relevance, which in turn helps make sense of the contrast in IIs between (1) vs. (2). In both cases, higher-numbered alternatives are OI because the sentence doesn't settle or entail ignorance about them (in any context). The difference between (1) and (2) is that in the former case, $[= 2]$ and $[> 2]$ can both be relevant, because when they are, (1) entails $K[\geq 2] \wedge \neg K[= 2] \wedge \neg K[> 2]$, which entails ignorance about $[= 2]$ and $[> 2]$. This result is a consequence of the symmetry between $[= 2]$ and $[> 2]$. In the case of (2), no symmetry arises (e.g. between $[= 3]$ and $[> 3]$, relative to $[> 2]$) due crucially to the UDM. Thus, density and symmetry play an important and symbiotic role in the theory under consideration.

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