Explaining ignorance inferences and roundness effects of modified numerals

We propose a unified account for three puzzles concerning modified numerals (e.g., *more than n*). [A] Superlative modifiers (*at least/at most*) lead to much stronger ignorance inferences than comparative modifiers (*more than/less than*) [GN07, a.o.]. [B] The distribution of modified numerals depends on the question under discussion (QUD); in particular, *more than n* is rarely used in response to a fine-grained *how many* QUD [WB14, Eng18]. [C] The roundness and contextual salience of the numeral involved plays an important role in determining the acceptability of comparative (though not superlative) modified numerals [CSS12, WB14, Eng18], as illustrated in (1):

(1) Mary can drink, she's at least 27/*more than 27 years old. [*legal drinking age: 21*]

Existing accounts: [Cum11] offers an Optimality Theory (OT) account of [C], but it wrongly predicts that *at least* should be just as sensitive to roundness as *more than*. [A] is addressed by separately stipulating that *at least* triggers stronger ignorance inferences, and [B] is not addressed in detail.

[WB14] offer an account of [A], based on the assumption that *at least* is typically used to address precise *how many* QUDs, while *more than* is typically used in contexts where the exact quantity does not matter (this assumption corresponds to observation [B]). They justify this assumption with a corpus study showing that *more than* is used much more often with round numerals than with non-round numerals, while *at least* does not show such a strong preference. However, this corpus finding, which is plausibly closely related to the effects of roundness on acceptability (observation [C] above), remains unexplained.

[Eng18] partly addresses [B] and [C], proposing an account of the roundness sensitivity of *more than* based on mandatory irrelevance implicatures. In short, the idea is that *more than n* triggers the inference that numerals directly above *n* are irrelevant, hence the incompatibility with precise *how many* QUDs and the preference for a salient or round *n*. This, however, does not capture the contrast between *more than* and *at least*.

Our proposal: Building on [Cum11], we offer a new OT account which derives [A-C] from a ranked set of general pragmatic constraints. Given a triplet $\langle \varphi, s, Q \rangle$ consisting of an expression φ, a speaker's information state $s \subseteq W$, and a QUD Q (a partition of W), we assume the following constraints. Quality (QUAL) requires that s supports φ ($s \subseteq [\![\varphi]\!]$). Quantity (QUANT) requires that φ resolves the QUD just as well as s (i.e., s should not exclude more Q-cells than φ). Numeral salience (NSAL), a markedness constraint adapted from [Cum11], is violated if φ contains a numeral that is neither round nor contextually salient. Internal salience (ISAL), a new faithfulness constraint, is violated if φ contains a numeral that is not internally salient to the speaker in the sense that it does not match a boundary of the range of values that the speaker considers possible. For instance, if a speaker believes that between 6 and 10 students left, then the expressions *n / at least n / more than n students left* satisfy ISAL just in case *n* is 6 or 10. ISAL could reflect a general pressure to align internal and external salience, which may be related to Quality/Quantity (communicate exactly what you know). Finally, Complexity (COMPL) penalizes complex expressions. In line with [Cum11] and the processing literature on modified numerals [e.g., CK10], we assume that *at least* incurs two violations, *more than* incurs one, and bare numerals none. We assume the following ranking of the constraints:

(2) QUAL $\gg$ QUANT $\gg$ NSAL $\approx$ ISAL $\gg$ COMPL

Following [Boe97], among others, we interpret $\approx$ in a probabilistic manner: if the constraints NSAL and ISAL are in conflict, they do not cancel each other but either of them could take precedence at evaluation time. Finally, we assume the usual naive semantics for modified numerals (*at least n P Q* is interpreted as $|P \cap Q| \geq n$, *more than n P Q* as $|P \cap Q| > n$) and an exact semantics for numerals (the account can also easily accommodate an ambiguity theory for bare numerals, provided an extra constraint governing the use of ambiguous expressions).

Predictions: Without going into too much details, the key predictions of the account are the following. Because *at least* incurs more violations of COMPL than *more than* and bare numerals, we predict that it can only be optimal if (i) competing bare numerals do not satisfy QUAL and (ii) *at least* satisfies ISAL. (i) means that *at least* is unacceptable if the speaker has precise knowledge, and (ii) means that *at least n* requires the speaker to consider *n* possible (otherwise *n* would not be the lower bound of the speaker's information state). Together, (i) and (ii) capture the behavior of *at least n*: the speaker must be ignorant and must consider both exactly *n* and more than *n* possible (observation [A]).

In answering fine-grained how-many QUDs, the speaker should always use the correct bare numeral when they have precise knowledge and always use *at least* when they only know a range. *More than* is out because, unlike *at least*, it cannot satisfy QUANT without violating ISAL (with some rare exceptions). This captures observation [B]. With polar or coarse-grained QUDs, bare numerals and *at least* remain options to convey the speaker's exact information state (precise or range, respectively), but *more than* becomes a viable alternative to both. Indeed, using the less informative *more than 50* to convey 53 does not violate QUANT if the QUD does not distinguish 50 from 53, so the bare numeral *53* and *more than 50* end up in a tie (the former satisfies ISAL but not NSAL, the latter NSAL but not ISAL). The same happens between a specific *at least* and a round *more than* in case of imprecise knowledge. More generally, because of the systematic violation of ISAL, we predict that *more than* must at least satisfy NSAL to stand a chance, which means that it must combine with a round or salient numeral (observation [C]).

Flipping the OT tableaux yields predictions about comprehension rather than production (assuming participants are subrational and ignore alternative utterances), which align with the experimental findings of [WB14]: Ignorance inferences depend entirely on the QUD, and not on the choice of modifier. Thus, a small set of general pragmatic constraints can capture a wide range of empirical findings on modified numerals that have heretofore eluded a unified analysis, without requiring any ad-hoc semantic assumptions.

References

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